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GRAVITY SEPARATION OF CONCENTRATED BIDISPERSE SUSPENSIONS IN INCLINED PLATE SETTLERS

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Abstract—Sedimentation rates of concentrated bidisperse suspensions containing light and heavy particles in inclined channels have been measured for two different fluid–particle systems. The first system consists of spherical polystyrene and polymethyl methacrylate beads in a salt solution and the second consists of spherical polystyrene and glass beads in a sucrose solution. Settling experiments have been conducted in two different rectangular channels of 4 × 6 cm and 3 × 8 cm cross sections. The angle of inclination was varied from 0° to 30° from the vertical. The initial concentration of the light and heavy particles in the suspension was varied from 8 to 20 vol%. The influence of the *Boycott effect*, due to inclination, and the *fingering phenomena*, due to lateral segregation, on the separation of the light and heavy particle species within the bidisperse zone was evaluated. The separation rate was found to be significantly influenced by the angle of inclination and the total initial concentration of the suspension. The *Boycott effect* due to the increasing angle of inclination compliments the separation due to the *fingering phenomena* up to a certain angle. This limiting angle, beyond which the performance degrades, is found to be a function of total solids concentration.

Key Words: bidisperse sedimentation, inclined plate settlers, fingering phenomena

1. INTRODUCTION

Gravity separation of light and heavy particulate species from a suspension is a common industrial process that often requires large equipment especially when the sedimentation velocities of one or both species are small. Thus, there is a need to reduce the required retention time of such settling vessels.

Inclined plate settlers based on applications of the *Boycott effect* (Boycott 1920) and *fingering phenomena* (Whitmore 1955; Weiland & McPherson 1979) offer a potential means to enhance the separation of light and heavy particles contained in a suspension.

Davis & Acrivos (1985) provide a good summary of recent studies on the *Boycott effect*. The earliest explanation of this phenomenon, based entirely on kinematic arguments, was presented by Ponder (1925) and Nakamura & Kuroda (1937). It is normally referred to as the PNK model. Several empirical and geometric modifications to the PNK model have been suggested (Kinosita 1949; Pearce 1962; Graham & Lama 1963; Oliver & Jenson 1964; Vohra & Ghosh 1971; Zahavi & Rubin 1975). A more fundamental approach, based on a solution of the ensemble-averaged flow equations, has been used by several authors (Hill *et al.* 1977; Acrivos & Herbolzheimer 1979; Herbolzheimer & Acrivos 1981; Schneider 1982; Davis *et al.* 1982; Herbolzheimer 1983; Schaflinger 1985a, b). The effect of sedimentation of polydisperse suspensions in inclined channels was studied by Davis *et al.* (1982) and by Schaflinger (1985b).

In vertical containers, sedimentation of bidisperse suspensions containing particles of non-uniform size and/or density has been studied quite extensively. Two classes of sedimentation behavior have been identified for bidisperse suspensions.

The first class deals with low particle concentrations where no lateral inhomogeneities (fingers) exist. Modeling of such suspensions has been successfully achieved by incorporating the use of material balance for each settling region and the appropriate settling velocity–concentration relationship. Separation of bidisperse suspensions containing only heavy particles of different size or density has been studied by Richardson & Meikle (1961), Smith (1966), Lockett & Al-Habbooby (1973, 1974), Lockett & Bassoon (1979), Mirza & Richardson (1979), Masliyah (1979), Greenspan & Ungarish (1982), Selim *et al.* (1983) and Patwardhan & Tien (1985). This approach has been extended to bidisperse suspensions of *light* and *heavy* particles by Law *et al.* (1987).

The second class of bidisperse suspensions is that of *high* concentration suspensions where lateral inhomogeneities or *fingers* arise spontaneously from an initially homogeneous suspension. This phenomenon was first noticed by Whitmore (1955) and studied extensively by Weiland and his co-workers (Weiland & McPherson 1979; Fessas & Weiland 1981, 1982, 1984; Fessas 1983) who used flow visualization techniques, measured settling rate data and provided a generalized settling correlation. More recent work by Batchelor & Janse van Rensburg (1986) indicated that these inhomogeneities are not necessarily finger-like. They offer the first rational explanation for the formation of such structures. It is believed that they are due to the instability of the bidisperse suspension and to the growth of a bidisperse form of a concentration wave. Cox (1987) developed a theoretical explanation of the *fingering phenomena*.

The formation of the *finger-like* structures within the bidisperse suspension tends to enhance the falling and rising rates of the heavy and light particles, respectively. Such enhancement of course is limited to the period while both the light and heavy particles form the bidisperse phase. However, once the two species are disengaged, settling would proceed in accordance with the local concentration of the newly formed individual monodisperse light and heavy particle suspensions.

Batch separation of light and heavy bidisperse suspensions at *low* concentrations in *inclined* channels has been investigated by Law *et al.* (1988). At these low concentrations, flow visualization showed the presence of distinct zones and interfaces, but no lateral inhomogeneities. The PNK model was adapted quite successfully to model particle separation in such suspensions.

In the present study we focus on the batch sedimentation of concentrated bidisperse suspensions containing light and heavy particle species in inclined channels. To our knowledge, no work has been performed to date on gravity separation where the *Boycott effect* and *fingering phenomena* are present. Our objective is to examine whether these effects are additive over a range of initial concentration and angle of inclination.

2. EXPERIMENTAL

The batch sedimentation experiments were conducted in two different rectangular Plexiglas channels with cross sections of 4 × 6 cm and 3 × 8 cm. Each channel was mounted on a steel stand that could be inclined from 0° to 90° from the vertical. The stand was carefully leveled and angle graduations were marked using a cathetometer and a calibrated ruler. The initial vertical height of each suspension was kept constant at 36.3 cm. Two different suspensions, one containing mono-sized particles of polymethyl methacrylate (PMMA) and polystyrene (PS) beads dispersed in an aqueous sodium chloride solution and another containing glass spheres and PS spheres dispersed in an aqueous sucrose solution, were used. For both suspensions, the addition of approx. 0.05% Triton X-100 inhibited flocculation of the particles. Details of the suspensions and containers used are given in tables 1 and 2. Particles of uniform size and density were prepared

Table 1

Property	*Particle species*		*Suspending Fluid*
System 1	PS	PMMA	Aqueous NaCl
ρ(g/cm^3)	1.050	1.186	1.120
d_P(cm)	0.0241	0.0231	
μ(g/cm s)			0.0141
System 2	PS	Glass	Aqueous sucrose
ρ(g/cm^3)	1.050	2.835	1.230
d_P(cm)	0.0438	0.0133	
μ(g/cm s)			0.1543

Table 2. Geometrical dimensions

Geometry	Width (cm)	Depth (cm)	Height (cm)
Wide spacing	4	6	80
Narrow spacing	3	8	80

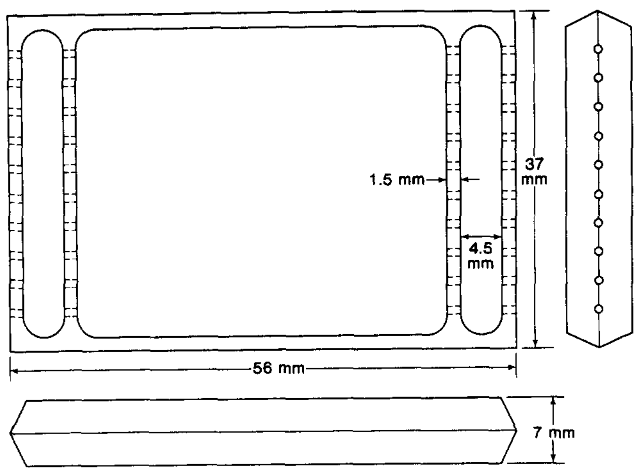

Figure 1. Schematic view of the diver for the 4 × 6 cm channel.

from the bulk sample by dry sieving followed by wet sieving. As a final step, suspensions of each species were allowed to settle in a long cylinder and any trailing particles were vacuumed off. This was repeated until a sharp interface was present throughout the settling period.

Particle diameters were measured from photomicrographs of 100 random samples using a Bausch & Lomb Omnimet image analyzer. The densities of the polymer particles were measured using the sink–float method outlined by Whitmore (1955). The density of the glass beads was measured using 50 ml specific gravity bottles.

Visual tracking of the settling heavy particles and rising light particles was not possible without the use of divers. A sketch of the diver is shown in figure 1. Two of these devices were included in each bidisperse suspension. The divers were machined from Jaytrex 1000 UHMW polyethylene, having a density of 0.942 g/ml. The leading edges of the divers were sharp so that particles would not rest upon the divers and change their effective density. The density of each diver was adjusted by adding or removing copper wires in such a manner that one would start at the top of the suspension and track downwards the falling heavy particles interface until disengagement and then track upwards the rising light particles interface. The density of the second diver allowed one to begin at the bottom of the suspension and track upwards the rising light particles interface until disengagement and then track downwards the falling heavy particles interface. When finger-like structures develop, clearly identifiable interfaces do not exist, as was pointed out by Fessas & Weiland (1982). In such cases the divers can be thought of as tracking a zone of constant (area-averaged) density rather than a sharp interface. The delay between the time the two divers come together in the middle and the visual observation of disengagement of the two species is one measure of the error in tracking the zones. This delay was minimized as much as possible. Such painstaking adjustments to the divers' density were necessary each time the initial concentration of the suspension was changed. Once a diver worked satisfactorily for a given initial concentration, it was kept the same for a series of runs at various angles of inclination. Thus, the measured enhancement rate due to the *Boycott effect* is less prone to error.

Each run was performed by first tilting the tube to the desired angle. The suspension was then homogenized by gentle stirring so as to minimize splashing of the particles on the tube walls above the suspension. One diver was quickly pulled to the top of the suspension with the stir-rod while the other diver was pushed to the bottom of the tube. The stir-rod was placed at the bottom of the tube and the timer sequence started on the computer. Height–time data were logged onto a personal computer until sedimentation was complete. On completion of one set of runs, the particles were separated, rinsed and dried for re-use. Solution samples were taken for measurement of solute concentration and solution density.

Figure 2 shows the details of typical settling curves of a bidisperse suspension at various angles. The initial portion of each curve reflects the accelerated settling period. This continued until the disengagement point where the two divers met and the rising and falling interfaces passed by one

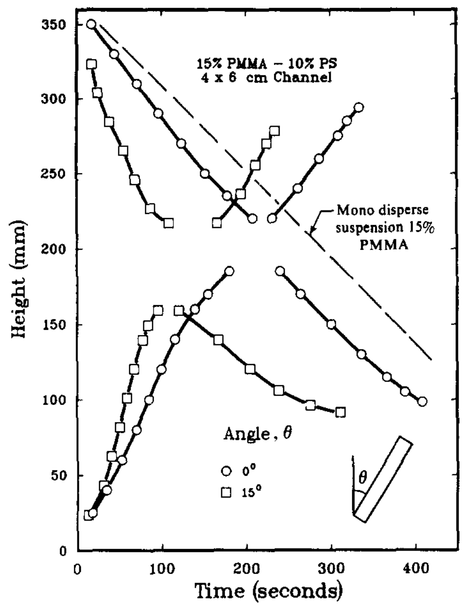

Figure 2. Settling of a bidisperse suspension.

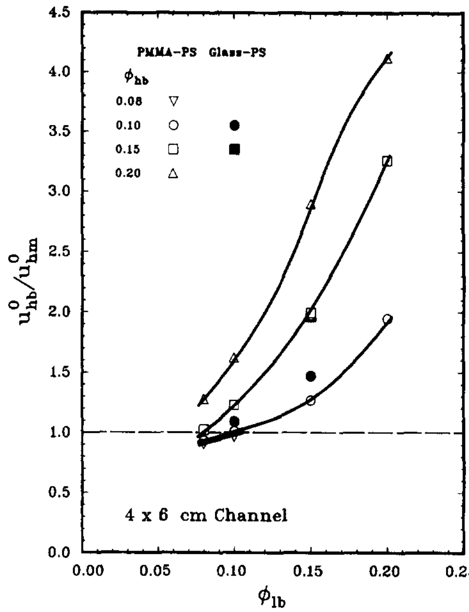

Figure 3. Variation of u^0_{hb}/u^0_{hm} with light particle concentration at various levels of heavy particle concentration.

another. The open area near the center of each settling curve reflects this period. The average slope of the portion of the settling curve before disengagement is used as a representative settling velocity of a species in a bidisperse suspension at a given angle of inclination. Consequently, the results of this study refer to the bidisperse zone and do not include the period after the disengagement of the two particle species.

3. RESULTS AND DISCUSSION

3.1. Vertical monodisperse suspensions

First, the settling velocities of monodisperse suspensions in a vertical container were measured since they serve as the reference velocities for determining the influence of the *fingering* and *Boycott* effects. The settling velocity–concentration data (or the flux curve) from several settling experiments were assembled for each particle species in the form suggested by Shannon *et al.* (1963):

$$u_{jm} = \sum_{i=0}^{4} A_{ij}\phi^i_{jm}, \qquad j = \mathrm{l, h}, \tag{1}$$

where u_{jm} and ϕ_{jm} are the settling velocities (cm/s) and concentrations (volume fraction), respectively, of a monodisperse suspension, j (j = l for *light* and j = h for *heavy*). The coefficients, A_{ij}, of the polynomials for the systems used in this study are given in table 3.

3.2. Vertical bidisperse suspensions

In order to assess the magnitude of the fingering phenomena, tests were done on bidisperse suspensions in a *vertical* channel. Figures 3 and 4 show the dimensionless sedimentation velocity vs particle concentration for the light and heavy particle species in bidisperse suspensions in a vertical channel. The normalized velocity in figure 3 is the settling velocity of the *heavy* particles in the *bidisperse* suspension divided by the settling velocity of the *heavy* particles in a *monodisperse*

Table 3

Particle	A_0	A_1	A_2	A_3	A_4
PS (240 μm)	−0.1110	0.3802	0.3042	−2.5368	2.5344
PMMA	0.1150	−0.4892	0.5547	0.2832	−0.6584
Glass	0.0997	−0.3990	0.3065	0.6319	−0.8368

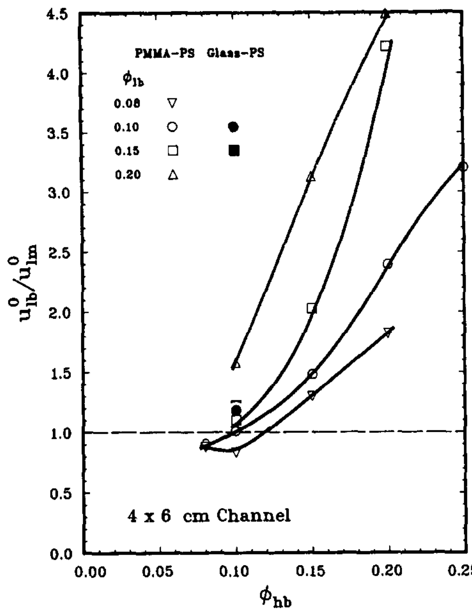

Figure 4. Variation of u^0_{lb}/u^0_{lm} with heavy particle concentration at various levels of light particle concentration.

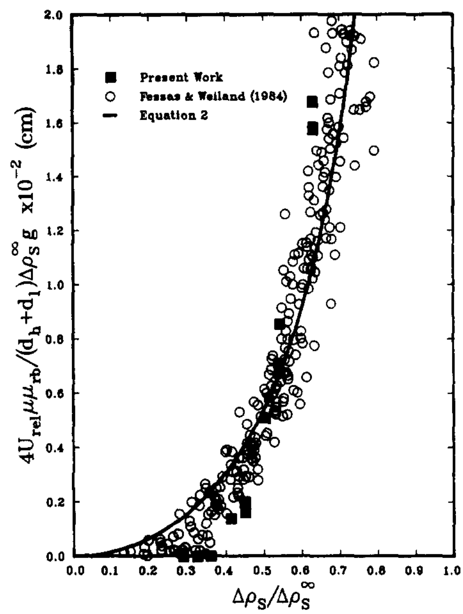

Figure 5. Comparison with previous studies.

suspension (obtained from [1]), at the same heavy particle concentration, i.e. $\phi_{hm} = \phi_{hb}$. The normalized settling velocity of the light particles in figure 4 is similarly defined. Henceforth u^θ_{hb} refers to the settling velocity of the heavy species in the bidisperse suspension at an angle of $\theta°$ from the vertical, while u^0_{hm} represents the settling velocity of heavy species in a monodisperse suspension at 0°. The settling velocity of a species in the *bidisperse* zone is obtained from the best estimate of the slope of the pre-disengagement portion of the height vs time curve, as shown in figure 2. In figures 3 and 4 enhancement of the settling velocity in the bidisperse zone above that of the corresponding monodisperse suspension is evident, except in cases of low concentrations. At low particle concentrations (< 16 vol. % of total solids) hindered settling results in minor retardation of the settling velocity. Law *et al.* (1987) found that the jump conditions across moving interfaces [derived from the one-dimensional continuity equation (Kynch 1952; Smith 1966)] could be combined with the solids flux relationship to predict the settling velocities of light and heavy particle species in bidisperse suspensions. At higher particle concentrations, figures 3 and 4 show a dramatic increase in the settling velocities. These curves show that the addition of the *enhancing* particles (light particles when examining heavy particles velocity and vice versa) causes a greater degree of enhancement of the settling velocity of the sedimenting particles when that species is present in the higher concentration. This is consistent with similar observations by Whitmore (1955) and Weiland & McPherson (1979). The threshold concentration, beyond which the settling rate begins to increase, appears to be system dependent to some degree. For the PMMA–PS–NaCl (aq.) system studied here and the glass–PVC–thallium formate system studied by Fessas & Weiland (1984) the threshold total solids concentration appears to be between 0.16 and 0.20. Also included in figures 3 and 4 are data from the glass–PS system. Although only a few experiments were done with this system, the degree of enhancement appears to be similar to the PMMA–PS system. Data for the PMMA–PS system in the 3 × 8 cm channel, although not shown in figures 3 and 4, were also found to agree well with the data from the 4 × 6 cm channel.

Fessas & Weiland (1984) developed a generalized correlation for predicting the enhanced settling rate due to the *fingering phenomena* by postulating a simple conceptual model. The particles present in the smaller concentration are postulated to move as *streams* or fingers through a *continuum* of the other species present in the larger concentration. They found the following functional form of the correlation to be adequate in representing the experimentally measured data. This correlation is useful as it consists entirely of experimentally measurable quantities, except for $(k'D)$ which is left as a model parameter, to be curve fitted:

$$\frac{4U_{\mathrm{rel}}\mu\mu_{\mathrm{rb}}}{(d_{\mathrm{s}}+d_{\mathrm{c}})\Delta\rho_{\mathrm{s}}^{\infty}g}=(k'D)f\left[\frac{\Delta\rho_{\mathrm{s}}}{\Delta\rho_{\mathrm{s}}^{\infty}}\right], \qquad [2]$$

where f is an empirical function, U_{rel} is the relative velocity, μ is the viscosity of the fluid, μ_{rb} is a normalized viscosity of the suspension, d_{s} and d_{c} are the particle diameters in the *stream* and the *continuum* respectively, D is the *stream* diameter, $\Delta\rho_{\mathrm{s}}$ is the density difference and $\Delta\rho_{\mathrm{s}}^{\infty}$ is the maximum (packed) density difference. The reader is referred to the original work of Fessas & Weiland (1984) for additional details on evaluating μ_{rb}, $\Delta\rho_{\mathrm{s}}$ etc. The data from the present investigation are presented in figure 5 along with those of Fessas & Weiland (1984) and the agreement is seen to be quite reasonable.

Recently, Batchelor & Janse van Rensburg (1986) mapped out sections of stability boundaries in the parameter space of concentrations, ϕ_{lb} and ϕ_{hb}, diameter ratio, $\lambda = d_{\mathrm{l}}/d_{\mathrm{h}}$, and reduced density ratio, $\gamma = [(\rho_{\mathrm{h}}-\rho_{\mathrm{f}})/(\rho_{\mathrm{l}}-\rho_{\mathrm{f}})]$. The subscripts l, h and f denote light, heavy and fluid, respectively. They defined stable systems as those in which no *coherent convective structures arose* and unstable systems as those were definite structures formed within a few seconds of the cessation of mixing. Marginally stable systems were those where no definite flow structure appeared, although there was some transient structure formation. Their own observations as well as most of the data from Fessas & Weiland (1981, 1984) provided a reasonably consistent picture of the stability boundary. In interpreting the data of Fessas & Weiland a dispersion was deemed unstable if the settling velocity was enhanced. When the results of figures 3 and 4 are interpreted in a similar manner the data from the present study was entirely consistent with those of previous studies.

3.3. Inclined monodisperse suspensions

Several experiments were performed with *monodispersed* suspensions in *inclined* channels, primarily for comparison with the *inclined bidisperse* experiments. Studies by Acrivos & Herbolzheimer (1979) indicate that the PNK model for a monodisperse system is valid when Λ, the ratio of the sedimentation Grashof number to the Reynolds number, approaches infinity and when the interface between the clear fluid and the suspension remains stable. For the systems used in this study Λ is

$$3.29\times10^{6}\leqslant\Lambda\leqslant1.03\times10^{7}.$$

Our experimental results for the monodisperse suspensions show a somewhat lower sedimentation rate than that predicted by the PNK model, which is known (Acrivos & Herbolzheimer 1979) to predict an upper bound to the sedimentation velocity. The PNK model, in its simplest form (i.e. without accounting for the sediment), is given below:

$$\frac{\mathrm{d}H}{\mathrm{d}t}=-u_{\mathrm{hm}}^{0}\left(1+\frac{H}{b}\sin\theta\right), \qquad [3]$$

where H is the height of the suspension, b is the spacing between the plates, θ, is the angle of inclination and u_{hm}^{0} is the vertical settling velocity of the heavy monodispersed phase. The second term gives a measure of the enhancement over the vertical settling velocity in a monodisperse system.

3.4. Inclined bidisperse suspensions

First, the overall influence of the combined mechanisms of the *Boycott effect* and the *fingering phenomena* are presented over a range of particle concentrations and angles of inclination. These velocities are normalized with respect to the corresponding monodisperse settling velocities in a vertical container at the same particle concentration as in the bidisperse system, i.e. $\phi_{\mathrm{hm}}=\phi_{\mathrm{hb}}$, for the heavy particle settling velocity. This is followed by the assessment of the magnitude of individual mechanisms in sections 3.4.2. and 3.4.3.

3.4.1. Overall effect. Figures 6a and 6b show the normalized settling velocity of the heavy species as a function of the angle of inclination, θ, for different amounts of light species (ϕ_{lb}) added to the system. The heavy species concentration is fixed at 0.10 and 0.15 in figures 6a and 6b,

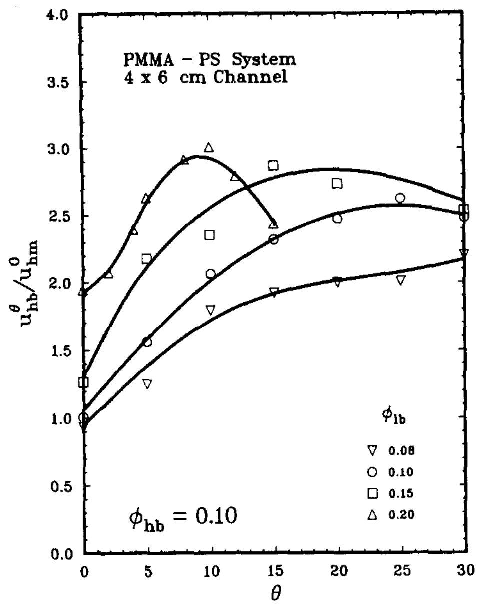

Figure 6a. Variation of u^θ_{hb}/u^0_{hm} with inclination angle, θ, at $\phi_{hb} = 0.10$ for the 4×6 cm channel.

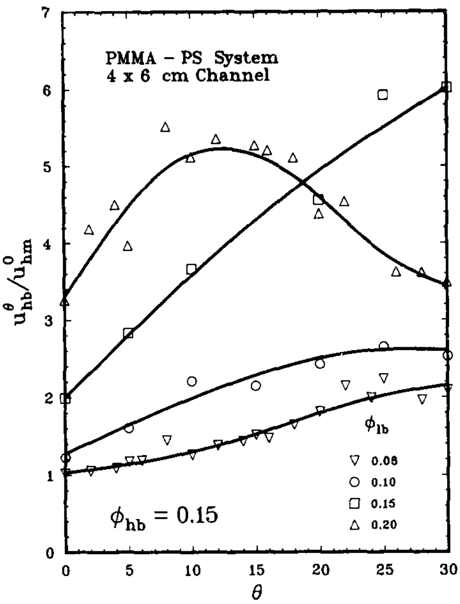

Figure 6b. Variation of u^θ_{hb}/u^0_{hm} with inclination angle, θ, at $\phi_{hb} = 0.15$ for the 4×6 cm channel.

respectively. Figures 7a and 7b show similar results for the light species ($\phi_{lb} = 0.10$, 0.15) with the heavy species serving as the enhancing particles. These results are for the 4×6 cm channel. For a *vertical* case ($\theta = 0°$) each of figures 6a and 6b show that the degree of enhancement increases with the addition of more enhancing light particles to the system. Comparison of figure 6a with 6b shows that the degree of enhancement is also higher when the original amount of the sedimenting heavy particles in the system is higher. Similar trends are observed in figures 7a and 7b, and these are entirely consistent with the findings of Fessas & Weiland (1981) for $\theta = 0°$. However, Fessas & Weiland (1981) indicated that there is an eventual degradation in performance once the total solids concentration exceeds 0.40.

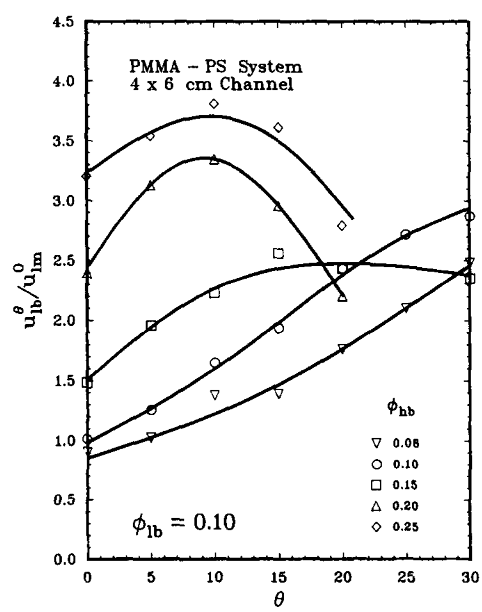

Figure 7a. Variation of u^θ_{lb}/u^0_{lm} with inclination angle, θ, at $\phi_{lb} = 0.10$ for the 4×6 cm channel.

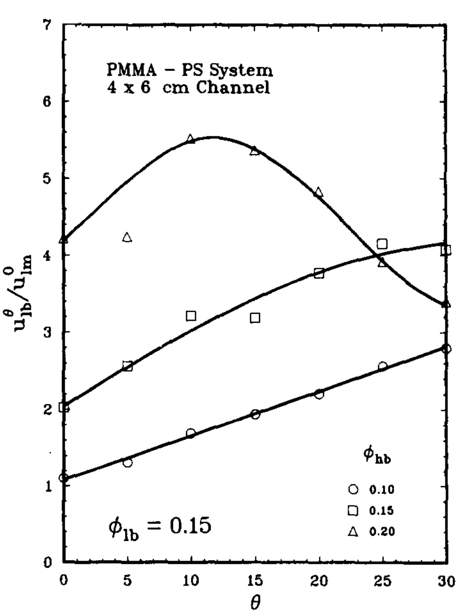

Figure 7b. Variation of u^θ_{lb}/u^0_{lm} with inclination angle, θ, at $\phi_{lb} = 0.15$ for the 4×6 cm channel.

At small angles of inclination (up to 10° from vertical) the *Boycott effect* compliments the settling process, thus increasing the settling rate with increasing angle of inclination. With a continued increase in the angle of inclination some settling curves (viz. those with the highest concentration of enhancing particles) reach a maximum enhancement and a performance degradation follows. The curves of 20% *enhancing* particles quite consistently give maxima between 10° and 15°. Curves of 15% *enhancing* particles generally tend to flatten out at between 20° and 30°. Curves of 8 and 10% *enhancing* particles either tend to flatten out or keep increasing within the range of angles of inclination tested here. Schaflinger (1985b) noticed that sedimentation of polydisperse suspensions under inclined walls at moderately high Reynolds number can lead to resuspension of particles. The thick sediments that are deposited along the inclined walls of the channel at high particle concentrations effectively reduce the width of the inclined channel. This effective narrowing of the plate spacing tends to promote re-entrainment of the sediment back into suspension. Both these effects become more pronounced at large angles and at high concentrations and they tend to reduce the overall sedimentation rate of the particles.

The effect of a narrow channel spacing on the settling velocity is shown in figure 8. Here the particle species are the same as in figures 6a,b and 7a,b, however the channel cross section has been changed from 4 cm wide × 6 cm deep to 3 × 8 cm. The width refers to the plate spacing, b, as given in [3]. The curves for 8, 10, 15 and 25% heavy particles in a suspension of 10% light particles in figure 8 remain qualitatively similar to those in figures 6a,b and 7a,b. However, with increasing angle of inclination the *Boycott effect* becomes stronger in a narrow channel, resulting in a significant increase in the settling rates. This is shown more clearly in figure 9 where the ratio of the normalized settling rates in the 3 × 8 cm channel to the normalized settling rates in the 4 × 6 cm channel is plotted against the angle of inclination. It is clear that at angles less than about 15°, the sedimentation rates in the two channels are very similar. At higher angles, the sedimentation rate increases more rapidly in the 3 × 8 cm channel than in the 4 × 6 cm channel.

The effect of a different fluid–particle system, viz. the glass–PS–aqueous sucrose system, in the 4 × 6 cm channel is shown in figure 10. When compared to the corresponding curve for the PMMA–PS–NaCl(aq.) system in the 4 × 6 cm channel, shown in figure 6a, the trends and magnitudes of each curve appear to be quite similar. The fingering phenomena are still evident, as indicated by the sedimentation rates > 1 in a *vertical* channel. There also appears to be a similar interaction between the fingering phenomena and the Boycott effect, as evidenced by the leveling off of the 15% light particle species curve.

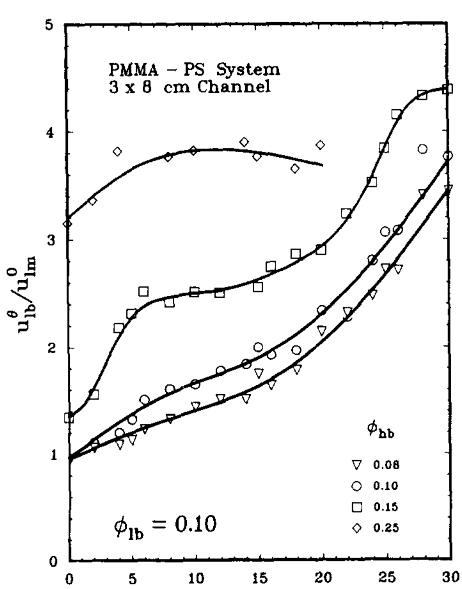

Figure 8. Variation of u^θ_{lb}/u^0_{lm} with inclination angle, θ, at $\phi_{lb} = 0.10$ for the 3 × 8 cm channel.

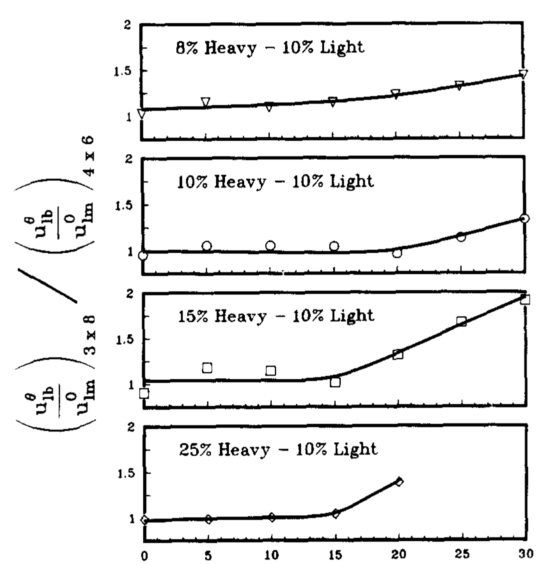

Figure 9. Effect of channel dimensions on settling velocities.

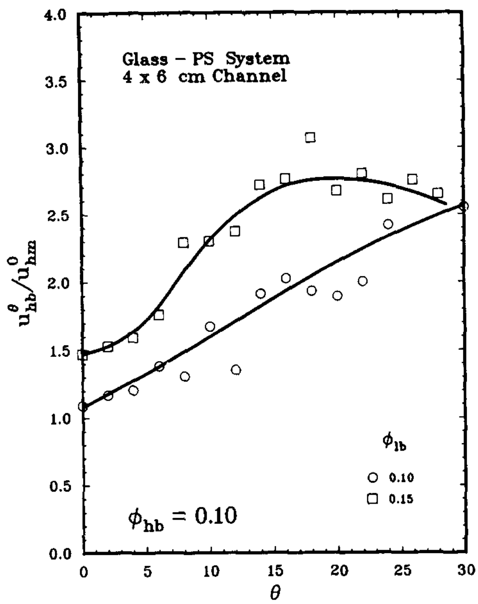

Figure 10. Variation of u^θ_{hb}/u^0_{hm} with inclination angle, θ, at $\phi_{hb} = 0.10$ for the glass = PS system in the 4 × 6 cm channel.

3.4.2. Fingering effect. In an effort to show the effect of angle of inclination on that portion of enhancement caused by the *fingering phenomena* alone, the settling velocities show in figures 6a,b and 7a,b are rescaled using the *monodispersed* settling rates in the same *inclined* channel and at the same sedimenting particle concentration. These results are shown in figures 11a,b and 12a,b. It must be pointed out that the scaling velocities used in figures 6a,b and 7a,b (viz. monodispersed–vertical geometry) are constant for each curve; but the scaling velocities used in figures 11a,b and 12a,b (viz. monodispersed–inclined geometry) change with the angle of inclination. The objective is to assess the influence of the addition of an enhancing particle in an inclined geometry. Once again the enhancement refers to that attained in the bidisperse zone. For smaller angles and wider

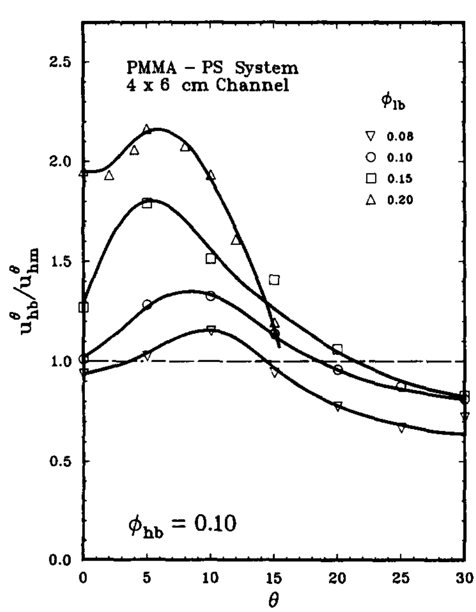

Figure 11a. Variation of $u^\theta_{hb}/u^\theta_{hm}$ with inclination angle, θ, at $\phi_{hb} = 0.10$ for the 4 × 6 cm channel.

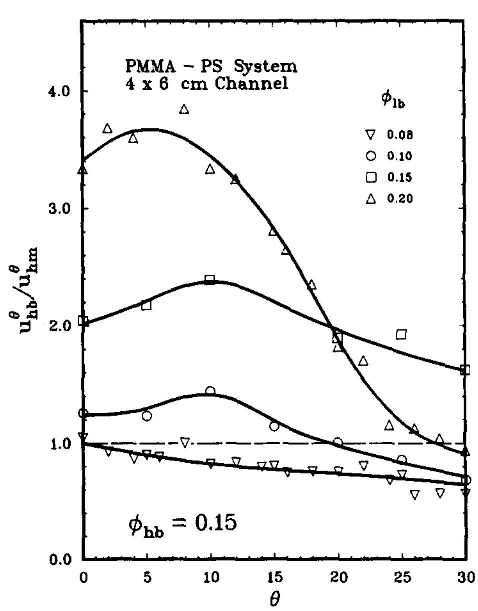

Figure 11b. Variation of $u^\theta_{hb}/u^\theta_{hm}$ with inclination angle, θ, at $\phi_{hb} = 0.15$ for the 4 × 6 cm channel.

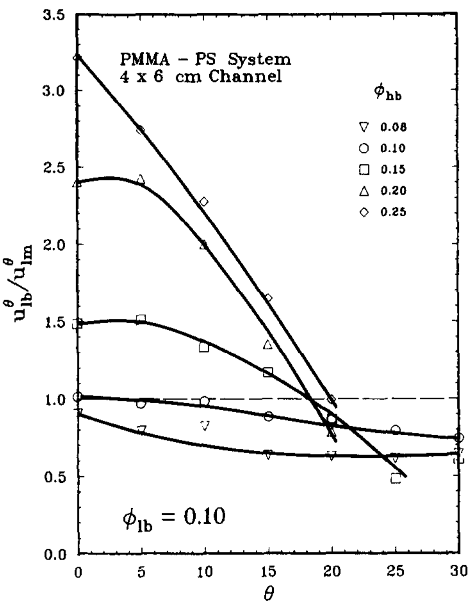

Figure 12a. Variation of $u^{\theta}_{lb}/u^{\theta}_{lm}$ with inclination angle, θ, at $\phi_{lb} = 0.10$ for the 4 × 6 cm channel.

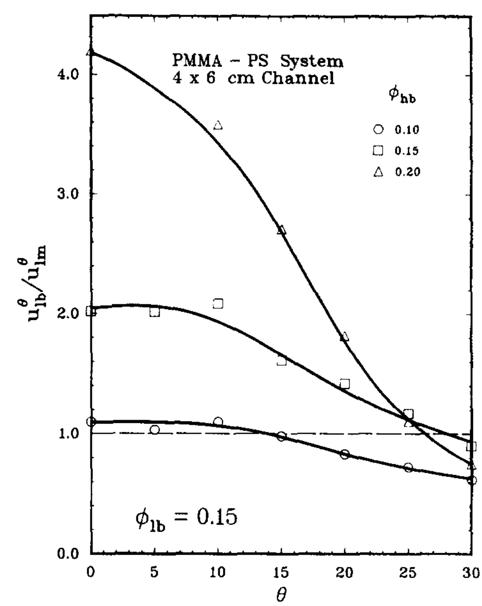

Figure 12b. Variation of $u^{\theta}_{lb}/u^{\theta}_{lm}$ with inclination angle, θ at $\phi_{lb} = 0.15$ for the 4 × 6 cm channel.

channels one can expect the *fingering phenomena* to be the controlling mechanism while at large angles and in narrow channels the *Boycott effect* should be in control. In the intermediate region the two mechanisms interact strongly with each other. The main feature evident in figures 11a,b and 12a,b is that the enhancement of the settling velocity caused by the *fingering phenomena* appears most significant at or near the vertical. As the angle of inclination increases beyond 5°–10°, or vertical in some cases, enhancement due to the *fingering phenomena* is steadily decreased until no enhancement due to the addition of the second particle species is evident somewhere between 20°–30°. In some cases retardation in the settling rate occurs, which is caused by the entrainment of particles. At large angles and/or small channel widths the *fingering phenomena* appears to play no role at all because the time scale for a particle species to be affected by the *Boycott effect* is much smaller than the time scale required for the formation of the fingering flow structure. The particle has to travel a shorter distance to be affected by boundary enhanced convection when the channel is inclined than when it is vertical.

It may appear that the greater the amount of enhancing particle species added to the system, the steeper the drop in the settling velocity. This is simply an artifact of the scaling process. The velocities are scaled in such a way that the enhancement factor should approach unity at large angles of inclination when the *Boycott effect* becomes controlling.

It should also be noted that if only a small amount of the *enhancing* particles is added to the system then little or no gain in the settling rate is achieved. In some cases, as in figure 12a with the addition of 8% *enhancing* particles, retardation of the settling rate occurs at *any* angle. This is consistent with earlier results as a threshold concentration must be exceeded before fingering flow is established.

Results of the *fingering phenomena* for the PMMA–PS–NaCl(aq.) system in the 3 × 8 cm, narrow channel are shown in figure 13. Notice that the scaling velocities of the monodisperse systems are also from the narrow channel at the same inclination and particle concentration. The limiting regions of the curves tend to be similar to those in figure 12a; but there is a strong interaction between the two mechanisms over the intermediate concentrations and angles. The curves corresponding to 8 and 10% of heavy *enhancing* particles in figure 13 show a significant difference. Note that bidisperse suspensions at these concentrations are only marginally stable, as observed by Batchelor & Janse van Rensburg (1986); hence strong fingering structures do not develop. Thus, the retardation observed up to an angle of 15° is due to the *waves* formed near the inclined plates as observed by Herbolzheimer (1983). These waves are formed near *both* plates in a *bidisperse*

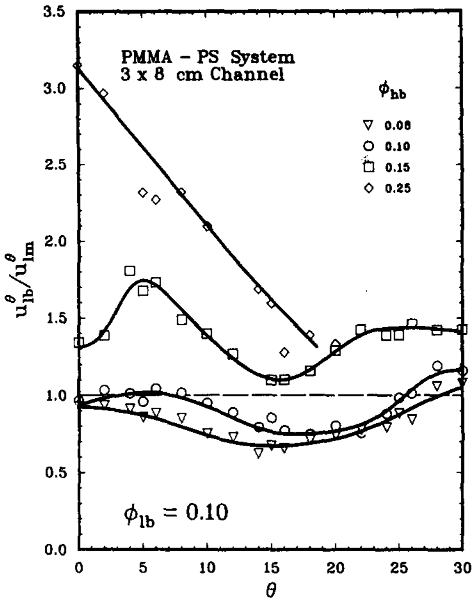

Figure 13. Variation of $u^{\theta}_{lb}/u^{\theta}_{lm}$ with inclination angle, θ, at $\phi_{lb} = 0.10$ for the 3×8 cm channel.

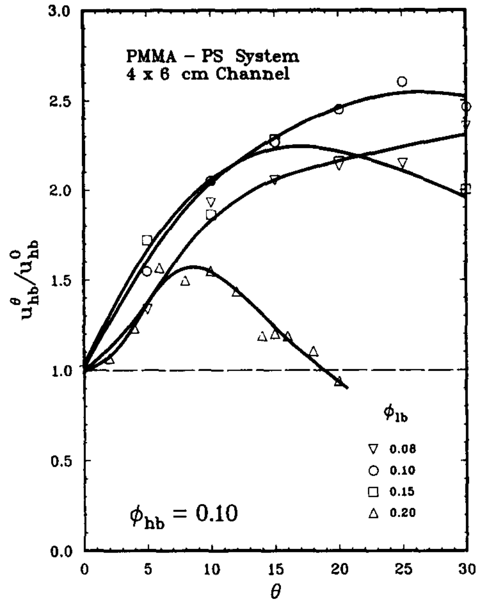

Figure 14a. Variation of $u^{\theta}_{hb}/u^{0}_{hb}$ with inclination angle, θ, at $\phi_{hb} = 0.10$ for the 4×6 cm channel.

system and interact strongly in a narrow channel, resulting in a retardation below that of the monodisperse system in the same inclined channel. These waves do not grow to their full extent at large angles (Herbolzheimer 1983) and hence the enhancement factors recover at larger angles.

3.4.3. Inclination effect. Next the effect of angle of inclination and the initial concentration on the Boycott effect is assessed by keeping the contribution of the *fingering phenomena* at about the same level. This is presented in figures 14a and 14b. The normalized velocity in figures 14a and 14b is the settling velocity of the sedimenting species in a bidisperse suspension at a *given angle* divided by the settling velocity of the same species in the same bidisperse suspension in a *vertical* channel. By keeping the concentration of the bidisperse suspension the same, the potential for

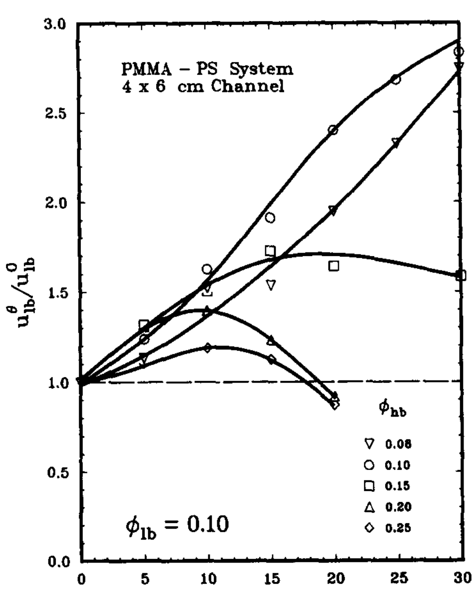

Figure 14b. Variation of $u^{\theta}_{lb}/u^{0}_{lb}$ with inclination angle, θ, at $\phi_{lb} = 0.10$ for the 4×6 cm channel.

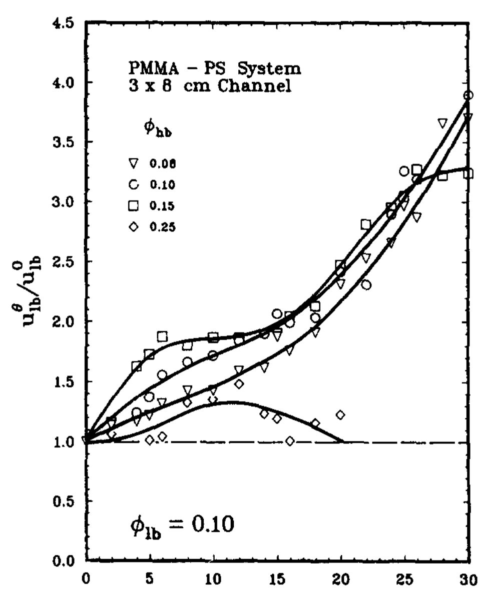

Figure 15. Variation of $u^{\theta}_{lb}/u^{0}_{lb}$ with inclination angle, θ at $\phi_{lb} = 0.10$ for the 3×8 cm channel.

enhancement due to the *fingering phenomena* is kept relatively the same. Unlike the previous case, the scaling velocity for each curve is a constant, but varies from one curve to another in such a way that all curves at $\theta = 0^\circ$ pass through $u^{\theta}_{jb}/u^{0}_{jb} = 1$, ($j$ = l or h).

The shapes of these curves are exactly the same as those in figures 6a and 7a, but their relative positions are changed. The largest increase in the sedimentation velocity due to the *Boycott effect* occurs at low concentrations (where the contribution due to the *fingering phenomena* is least). At higher concentrations there is an optimum angle beyond which the contributions from the two mechanisms are no longer additive. Even at the low concentration limit it is interesting to observe that the maximum increase occurs when the concentrations of both the particles are the same (i.e. 10% in this case).

Results similar to the above for the same fluid–particle system, but for the narrow 3 × 8 cm channel are presented in figure 15. The general trends are the same as in figures 14a and 14b, but the contribution from the *Boycott effect* is larger due to the narrow channel spacing. As in figure 8, this figure also shows very little enhancement due to the *Boycott effect* at the highest concentration (where clearly the fingering effect is dominant).

4. CONCLUSIONS

This work is the first step in analyzing the sedimentation behavior of *bidisperse* suspensions between *inclined* parallel plates. Although the two mechanisms are strongly coupled over certain ranges of inclination and concentration, some assessment of the individual contribution of each mechanism at comparable levels of the other mechanism has been made. The combination of the *fingering phenomena* and the *Boycott effect* was found to significantly influence the sedimentation rates. However, the coupling of the two effects results in an optimum combination of particle concentration and inclination angle to produce the maximum sedimentation velocity. A better understanding of the fundamental characteristics of the fingering flow structure is required in order to be able to predict the performance of such settling systems. It also remains to be seen whether the enhancement due to the *fingering phenomena*, as observed in the bidisperse zone, can be realized in a continuous mode of operation—as that depends on the ability to maintain a large stable bidisperse zone in a continuous settler.

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